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Constraint on the quadrupole moment of super-massive black hole candidates from the estimate of the mean radiative efficiency of AGN

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The super-massive objects at the center of many galaxies are commonly thought to be black holes. In 4-dimensional general relativity, a black hole is completely specified by its mass M and by its spin angular momentum J. All the higher multipole moments of the gravitational field depend in a very specific way on these two parameters. For instance, the mass quadrupole moment is $Q = -J^2/M$. If we can estimate M, J, and Q for the super-massive objects in galactic nuclei, we over-constrain the theory and we can test the black hole hypothesis. While there are many works studying how this can be done with future observations, in this paper a constraint on the quadrupole moment of these objects is obtained by using the current estimate of the mean radiative efficiency of AGN. In terms of the anomalous quadrupole moment q, the bound is $-2.01 < q < 0.14$.

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Introduction – Today we believe that the final product of the gravitational collapse is a black hole (BH) and we have robust observational evidences of the existence of $5-20$ Solar mass compact objects in X-ray binary systems [1] and of 10^5-10^9 Solar mass objects at the center of many galaxies [2]. All these objects are interpreted as BHs because they cannot be explained otherwise without introducing new physics. The stellar-mass objects in X-ray binary systems are too heavy to be neutron or quark stars for any reasonable equation of state [3]. At least some of the super-massive objects in galactic nuclei are too heavy, compact, and old to be clusters of non-luminous bodies [4]. However, there are no direct observational evidences that they have an event horizon [5], while there are theoretical arguments suggesting significant deviations from the classical picture [6].

In 4-dimensional general relativity, (uncharged) BHs are described by the Kerr solution and are completely specified by two parameters: the mass, M, and the spin angular momentum, J. The condition for the existence of the event horizon is $|a_*| \le 1$, where $a_* = J/M^2$ is the dimensionless spin parameter. The fact that a BH has only two degrees of freedom is known as "no-hair" theorem [7] and implies that all the mass moments, M_l, and all the current moments, S_l, of the gravitational field can be written in terms of M and J by the following simple formula:

$$M_l + iS_l = M\left(i\frac{J}{M}\right)^l . \qquad (1)$$

The first three non-trivial terms are the mass $M_0 = M$, the spin angular momentum $S_1 = J$, and the mass quadrupole moment $M_2 \equiv Q = -J^2/M$. On the contrary, for a generic compact object M_l and S_l can assume any arbitrary value, but in case of reflection symmetry, all the odd M_l-moments and all the even S_l-moments are identically zero. As it was put forward by Ryan in [8], by measuring the mass, the spin, and at least one more non-trivial moment of the gravitational field of a BH candidate, one over-constrains the theory and can test the Kerr BH hypothesis.

There is a whole line of research devoted to study how future experiments will be able to measure the mass quadrupole moment of BH candidates and thus test the nature of these objects. The most studied and promising approach is through the detection of gravitational waves of the inspiral of a stellar-mass compact object into a super-massive BH candidate [9]. Other proposals involve the observation of the BH shadow [10], the possible discovery of a stellar-mass BH candidate with a radio pulsar as companion [11], and accurate measurements of stellar orbits at mpc distances from Sgr A* [12]. There are also two proposals to constrain Q with current available X-ray data, by studying the Kα iron line [13] and the disk's thermal spectrum [14]. Only Ref. [14] constrains Q by considering the stellar-mass BH candidate M33 X-7, but the analysis is based on a simplified model and the bound is only meant as a qualitative guide for future more rigorous studies.

Radiative efficiency of AGN – The energy radiated by a compact object as a consequence of the accretion process is simply $L_{acc} = \eta \dot{M} c^2$, where η is the efficiency parameter, $\dot{M}$ is the mass accretion rate, and c is the speed of light. If the accreting gas cannot radiate efficiently its gravitational energy and the compact object is capable of absorbing quickly all the particles hitting its surface, η can be very small. For example, the efficiency parameter of the super-massive BH candidate in the Galaxy is estimated to be $\sim 5 \cdot 10^{-6}$ [15]. On the contrary, if all the gravitational energy is released as the gas sinks in the potential well of the compact object, $\eta = 1 - E_{\rm ISCO}$, where $E_{\rm ISCO}$ is the specific energy of the gas particles at the innermost stable circular orbit (ISCO) and depends on the metric of the space-time. For a Schwarzschild BH,

$\eta \approx 0.057$, while for a rotating BH η can be much higher, up to about 0.42.

If the distance from the compact object is known, L_{acc} can be easily measured. However, an accurate estimate of $\dot{M}$ is typically much more problematic and model dependent. It is instead possible to determine the mean efficiency parameter of active galactic nuclei (AGN) [16]. From the observed hard diffuse X-ray background and a quasar spectral energy distribution, one can estimate u_γ, the total contribution of quasar luminosity to the mean energy density of the Universe. From the study of the super-massive BH candidates in nearby galaxies, one can estimate ρ_{BH}, the mean mass density of BHs in the contemporary Universe. Under the conservative assumption that these objects acquire most of their mass through the accretion process, one divides u_γ by ρ_{BH}, to obtain an estimate of the average accretion efficiency η. Current studies find $\eta > 0.15$ [17, 18]. There are several uncertainties in this value, but 0.15 seems to be a reliable lower bound, especially for the most massive systems, because it is obtained from a set of conservative assumptions. An average efficiency around $0.30 - 0.35$ seems to be a reasonable estimate [18]. $\eta > 0.15$ is possible for a rapidly rotating BH with $a_* > 0.89$.

Compact objects with non-Kerr quadrupole moment – The Manko-Novikov (MN) metric is a stationary, axisymmetric, and asymptotically flat exact solution of Einstein's vacuum equation [19]. It is not a BH solution, but it can be used to describe the space-time around a compact body with arbitrary mass multipole moments. The solution has an infinite number of free parameters and the full expression can be seen in Ref. [14], where a few typos present in the original paper were corrected. Here I consider a subclass of the MN metric, with only three free parameters: the mass M, the spin parameter a_*, and the anomalous quadrupole moment q. The latter is defined by

$$Q = Q_{\rm Kerr} - qM^3 \,, \tag{2}$$

where $Q_{\rm Kerr} = -a_*^2 M^3$ is the mass quadrupole moment of a BH. For $q = 0$, we recover exactly the Kerr metric, while for $q > 0$ ($q < 0$) the object is more oblate (prolate) than a BH. The MN solution is written in prolate spheroidal coordinates and requires $|a_*| < 1$, even if this is not a fundamental limit as in the BH case. However, at least for small deviations from the Kerr metric, compact objects with $a_* > 1$ should be unstable [20].

The efficiency parameter η can be computed as follows. As in any stationary and axisymmetric space-time, the geodesic motion in cylindrical coordinates (t, r, z, ϕ) is governed by the following equations

$$\dot{t} = \frac{E g_{\phi\phi} + L_z g_{t\phi}}{g_{t\phi}^2 - g_{tt} g_{\phi\phi}} \,, \tag{3}$$

$$\dot{\phi} = -\frac{E g_{t\phi} + L_z g_{tt}}{g_{t\phi}^2 - g_{tt} g_{\phi\phi}} \,, \tag{4}$$

$$g_{rr}\dot{r}^2 + g_{zz}\dot{z}^2 = V_{\rm eff}(E, L_z, r, z) \,, \tag{5}$$

where E and L_z are respectively the conserved specific energy and the conserved specific z-component of the angular moment, while $V_{\rm eff}$ is the effective potential

$$V_{\rm eff} = \frac{E^2 g_{\phi\phi} + 2EL_z g_{t\phi} + L_z^2 g_{tt}}{g_{t\phi}^2 - g_{tt} g_{\phi\phi}} - 1 \,. \tag{6}$$

Circular orbits in the equatorial plane are located at the zeros and the turning points of the effective potential: $\dot{r} = \dot{z} = 0$ implies $V_{\rm eff} = 0$, and $\ddot{r} = \ddot{z} = 0$ requires $\partial_r V_{\rm eff} = \partial_z V_{\rm eff} = 0$. The specific energy turns out to be

$$E = -\frac{g_{tt} + g_{t\phi}\Omega}{\sqrt{-g_{tt} - 2g_{t\phi}\Omega - g_{\phi\phi}\Omega^2}} \,, \tag{7}$$

where

$$\Omega = \frac{d\phi}{dt} = \frac{-\partial_r g_{t\phi} \pm \sqrt{(\partial_r g_{t\phi})^2 - (\partial_r g_{tt})(\partial_r g_{\phi\phi})}}{\partial_r g_{\phi\phi}} \tag{8}$$

is the orbital angular velocity and the sign $+$ $(-)$ is for corotating (counterrotating) orbits. The orbits are stable under small perturbations if $\partial_r^2 V_{\rm eff} \le 0$ and $\partial_z^2 V_{\rm eff} \le 0$. In this way, one determines the specific energy at the inner radius of the disk, E_{in}, and the efficiency parameter $\eta = 1 - E_{in}$ for a particular choice of the spin parameter a_* and of the anomalous quadrupole moment q[1], see Fig. 1.

As the mean efficiency parameter of AGN must be larger than 0.15, one can constrain the mean spin and the mean quadrupole moment of these objects. This is done in Fig. 2, where the red curve denotes the boundary between the regions with $\eta > 0.15$ and $\eta < 0.15$. The constrain on the anomalous quadrupole moment q is

$$-2.01 < q < 0.14 \,. \tag{9}$$

If we adopt a more stringent bound on η, like $\eta > 0.20$, the constraint on q becomes $-0.96 < q < 0.03$.

Discussion – Eq. (9) provides a constraint on possible deviations from the Kerr metric around the super-massive BH candidates. The bound is much weaker for negative values of q because in these space-times either the inner radius of the disk and the specific energy at a given radius are usually smaller than the cases with $q > 0$. It is clear that the super-massive BH candidates must be objects very different from a compact body made of ordinary matter. For instance, the quadrupole moment of a neutron star is thought to be well approximated by the following expression

$$Q = -(1 + \tilde{q}) a_*^2 M^3 \,, \tag{10}$$

[1] In the MN space-times, for some $q < 0$ one finds two disconnected regions with stable circular orbits: one closer to the object, $r_1 < r < r_2$, and another for larger radii, $r > r_3$ with $r_3 > r_2$. The inner radius of the disk is r_3, as the orbits in the region $r_1 < r < r_2$ have larger energy and angular momentum [14].

with $\tilde{q} \approx 1 - 10$ independent of a_*, according to the matter equation of state and the mass of the body [21].

The constraint in Eq. (9) relies on the assumption that the mass of these objects is conserved during mergers. While this is a reasonable approximation for BHs in general relativity, we cannot say anything in the case of compact objects with unknown internal structure. If a substantial fraction of their mass were lost during merger, for instance through the emission of gravitational waves, the bound would be weaker, as the energy radiated in the accretion process would come from a larger amount of accreted mass. To obtain Eq. (9), I also assumed that the disk is on the equatorial plane. As explained in [22], this is justified by the fact that the timescale of the alignment of the spin of the object with the disk is much shorter than the time for the mass of these objects to increase significantly.

I would like to warn the reader that the estimate of η in Fig. 1 and the allowed region in Fig. 2 for the spin and the anomalous quadrupole moment inevitably partially depend even on the higher order moments of the spacetime. The latter are less and less important, but they are not completely negligible. This can be easily understood by noticing the difference in the constraint on the spin parameter a_* between a BH with $q = 0$ and a generic object with $q \neq 0$. For a BH, an efficiency parameter η larger than 0.15 requires $a_* > 0.89$. For $q \neq 0$, this bound relaxes to $a_* > 0.30$, see Fig. 2. This problem is present in any estimate of a quadrupole moment and therefore the future comparison of two limits on q obtained from different arguments or with different metrics deserves some attention.

Conclusions – There are not yet direct observational evidences that the super-massive objects at the center of many galaxies are the BHs predicted by general relativity, while recent theoretical arguments suggest that the final product of the gravitational collapse of matter may be quite different from what it is usually thought [6]. The BH hypothesis can be tested by measuring at least three non-trivial moments of the gravitational field of these objects, as in the case of a BH all the moments depend on the mass M and the spin J in a very specific way. There are several works in the literature discussing how this is possible with future experiments, but so far there are no constraints on the nature of these objects. For example, the future gravitational wave detector LISA will be able to measure the quadrupole moment of the super-massive BH candidates with a precision at the level of $10^{-2} - 10^{-4}$ (see the third paper in [9]). In this letter, I considered the current estimate of the mean radiative efficiency of AGN and I was able to constrain the anomalous quadrupole moment q of these objects. The bound I obtained is $-2.01 < q < 0.14$.

Lastly, let us notice that the maximum radiative efficiency for a BH is $\eta \approx 0.42$ when $a_* = 1$. A very fast-rotating object with q a little bit smaller than 0 can have a higher efficiency parameter. This implies, at least in principle, that the argument used in this paper may also rule out the Kerr BH hypothesis in the case of the discovery of an object with an efficiency parameter larger than the one that can be expected for a BH.

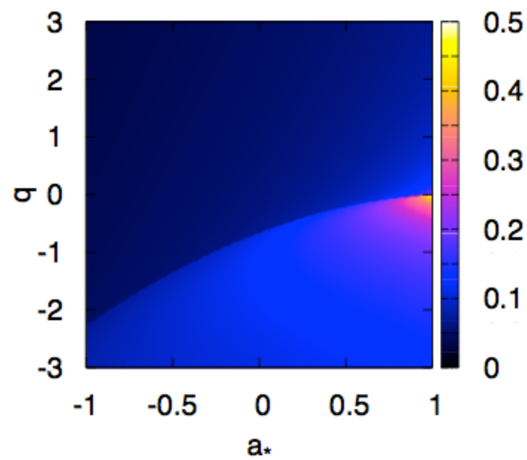

FIG. 1. Efficiency parameter η in the plane (a_*, q).

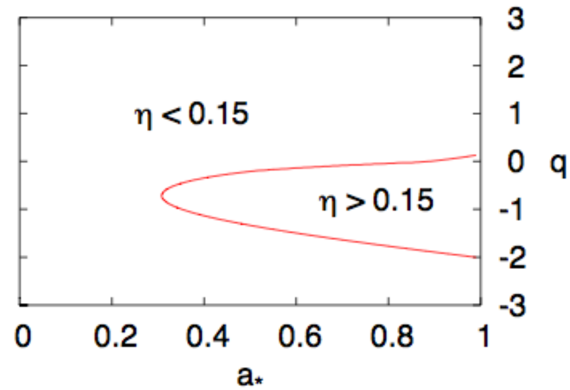

FIG. 2. Constraint on the mean spin parameter and the mean anomalous quadrupole moment of AGN from the estimate of their radiative efficiency. The allowed region is the one with $\eta > 0.15$.

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